Circular Motion

Suppose that an object moves around a circle of radius r at a constant speed. Then the speed v of the object, sometimes called *linear speed*, is the total distance traveled s divided by the elapsed time t.

$$v = \frac{s}{t} \tag{1}$$

Now let θ be the angle swept out in time t as the object moves around the circle. Then the *angular speed* ω of the object is defined to be the angle θ, measured in radians, divided by the elapsed time t.

$$\omega = \frac{\theta}{t} \tag{2}$$

Angular speed is often described in terms of revolutions per unit time, such as 600 rpm (revolutions per minute). Since 1 revolution = 2π radians, the angular speed of such an object is

$$600\,\frac{\text{revolutions}}{\text{minute}} = 600\,\frac{\text{revolutions}}{\text{minute}} \cdot 2\pi \frac{\text{radians}}{\text{revolution}} = 1200\,\pi \frac{\text{radians}}{\text{minute}}$$

If the object moves along an arc of length s, then the relationship $s = r\theta$ between arc length and radian measure (given in section 5.3) yields a corresponding relationship between linear and angular speed:

$$v = \frac{s}{t} = r\frac{\theta}{t} = r\omega \tag{3}$$

Linear speed has the dimension of length per unit of time, such as feet per second or miles per hour. Since radians are unitless and r has the dimension of length, notice that the right side of equation (3) also has the correct dimension of length per unit of time.

Circular Motion – Examples

1. Suppose the wheels on your bicycle are 28 inches in diameter, and you are traveling at a speed of 10 miles per hour. How many revolutions per minute are the wheels turning?

Solution: First convert speed to inches per minute:

$$\begin{aligned} v &= 10\,\frac{\text{miles}}{\text{hour}} \cdot 5280\,\frac{\text{feet}}{\text{mile}} \cdot 12\,\frac{\text{inches}}{\text{foot}} = 633,600\,\frac{\text{inches}}{\text{hour}} \\ &= 633,600\,\frac{\text{inches}}{\text{hour}} \cdot \frac{1}{60}\,\frac{\text{hours}}{\text{minute}} \\ &\approx 10,560\,\frac{\text{inches}}{\text{minute}} \end{aligned}$$

Now the radius of each wheel is $r = 14$ inches, so $v = r\omega$ from equation (3) implies

$$\omega = v/r \approx \frac{10,560}{14}\ \text{radians/minute} \approx 754.3\ \text{radians/minute}$$

Finally, converting to revolutions per minute yields

$$754.3\,\frac{\text{radians}}{\text{minute}} \cdot \frac{1}{2\pi}\,\frac{\text{revolutions}}{\text{radian}} \approx 120\,\frac{\text{revolutions}}{\text{minute}}$$

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2. Suppose that you are competing in the hammer throw, which consists of spinning a weight on the end of a rope and then releasing it. If the rope is 2.5 feet long and you spin the weight at a rate of 120 revolutions/minute, how fast does the weight travel when it is released?

Solution: First convert to radians per minute:

$$120\,\frac{\text{revolutions}}{\text{minute}} \cdot 2\pi\frac{\text{radians}}{\text{revolution}} = 240\pi\frac{\text{radians}}{\text{minute}} = \omega$$

Now using equation (3),

$$v = r\omega = 2.5\text{feet} \cdot 240\pi\frac{\text{radians}}{\text{minute}} \approx 1885\,\frac{\text{feet}}{\text{minute}}$$

or equivalently, ≈ 21.4 miles/hour.

Circular Motion Exercises

1. How many revolutions per minute are required to produce a hammer throw traveling at 30 miles per hour? As in example (2), assume that the rope is 2.5 feet long.

Solution: First convert speed to feet per minute:
$v = 30$ miles/hour $= 30 \cdot 5280/60$ feet/minute $= 2640$ feet/minute.
Now $r = 2.5$ feet, so $v = r\omega$ implies $\omega = v/r$
$= 2640/2.5$ radians/minute$= 1056$ radians/minute.
Finally, converting to revolutions per minute yields
$1056/(2\pi)$ revolutions/minute ≈ 168 revolutions/minute.

2. Suppose the wheels on your bicycle are 26 inches in diameter, and the wheels are turning at 4 revolutions per second. How fast are you traveling?

Solution: First find the angular speed in radians per minute:
4 revolutions/second$= 4 \cdot 2\pi \cdot 60$ radians/minute
$= 480\pi$ radians/minute$= \omega$.
Now $v = r\omega = 13$ inches $\cdot 480\pi$ inches/minute $\approx 19,604$ inches/minute, or equivalently, ≈ 18.56 miles/hour.

3. The cable on a chair lift at a ski resort is driven by a 6 foot diameter pulley that rotates at a speed of 30 revolutions per minute. How fast is each chair on the chair lift moving?

Solution: First find the angular speed in radians per minute:
30 revolutions/minute$= 30 \cdot 2\pi$ radians/minute
$= 60\pi$ radians/minute$= \omega$.
Now $v = r\omega = 3$ feet $\cdot 60\pi$ feet/minute ≈ 565.5 feet/minute,
or equivalently, ≈ 6.4 miles/hour.

4. Suppose the wheels on your car are 16 inches in diameter, and you are traveling at a speed of 60 miles per hour. How many revolutions per minute are the wheels turning?

Solution: First convert speed to inches per minute:
$v = 60$ miles/hour $= 60 \cdot 5280 \cdot 12/60$ inches/hour
$= 63,360$ inches/minute.
Now $r = 8$ inches, so $v = r\omega$ implies $\omega = v/r$
$= 63,360/8$ radians/minute$= 7920$ radians/minute.
Finally, converting to revolutions per minute yields
$7920/(2\pi)$ revolutions/minute ≈ 1260.5 revolutions/minute.