Math 142
Fall 1999

PRINT YOUR NAME: ______________________

SIGN YOUR NAME: ______________________

Analytic Geometry Final Exam

SECTION #: __________

For problems 1-11, show all your work, and write your answer in the blank provided. Each problem is worth 6 points. You can earn 0, 3, or 6 points on each problem. **Sufficient work must be shown to receive credit**.

1. Convert the polar coordinates $(6, \frac{2\pi}{3})$ to rectangular coordinates.

1. __________

2. Find the directrix of the parabola $y^2 = -6x$.

2. __________

3. Suppose $z_1 = 4e^{-\frac{\pi i}{6}}$ and $z_2 = 2e^{\frac{2\pi i}{3}}$.
Compute $\frac{z_1}{z_2}$ and express your answer in polar form.

3. __________

4. Find the foci of the conic section $x^2 - \frac{y^2}{9} = 1$.

4. __________

5. Convert the parametric equations $x = 2t$, $y = t^2 - 1$, to an equation in x and y only.

5. ____________

6. Find the length of the minor axis of the ellipse with center $(0, 0)$, focus $(0, 3)$, and vertex $(0, 5)$.

6. ____________

7. Find an appropriate first quadrant angle θ (in radians) so that a rotation of axes by θ transforms the equation $4x^2 + 2\sqrt{3}xy + 2y^2 = 25$ into a new equation of the form $au^2 + cv^2 + du + ev + f = 0$.
(You just need to find θ, *not* the new equation.)

7. ____________

8. Find the vertex of the parabola $x^2 - 4x = 2y$.

8. ____________

9. (a) Express the complex number $1 + i$ in polar form.

9.(a) ____________

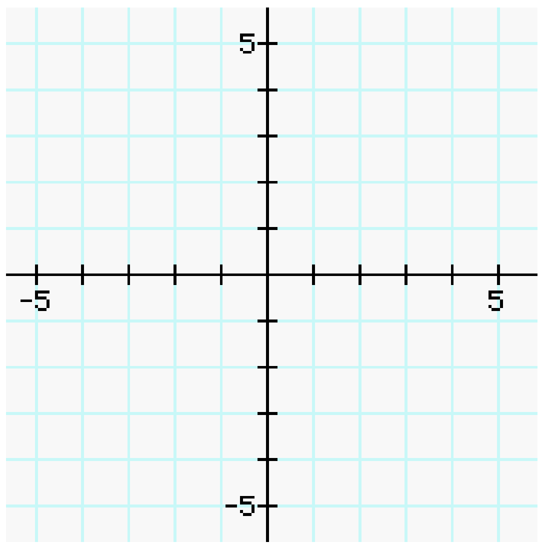
(b) Compute $(1 + i)^{20}$ and express your answer in standard form $a + bi$. Be sure to show your work.

(b) ____________

10. Convert the equation $r = 2$ to an equation in rectangular coordinates.

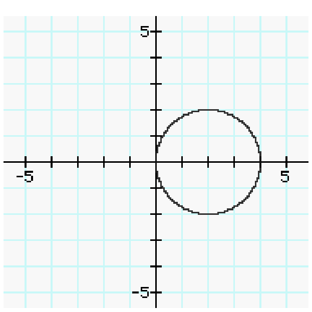
10. ____________

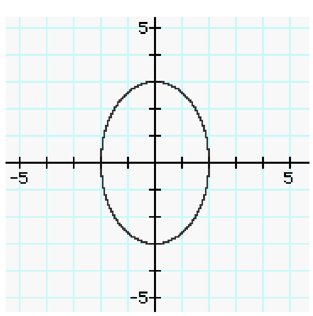
11. A rotation of axes by the angle $\theta = \frac{\pi}{4}$ transforms the equation $x^2 + 2xy + y^2 + 6\sqrt{2}x - 6\sqrt{2}y = 0$ into the equation $u^2 = 6v$. Sketch this conic section, showing the rotation (i.e., draw and label the u and v axes, and draw the conic section).

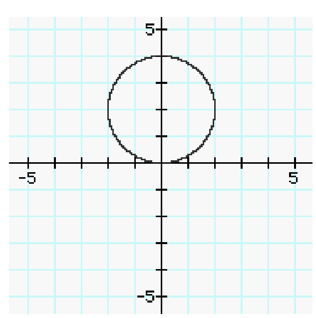


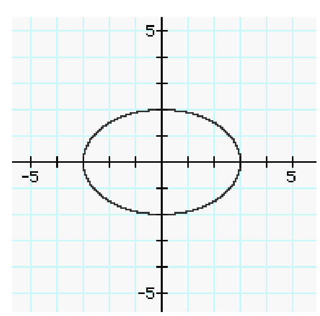
For problems 12 and 13, match the graphs with their corresponding equations. Write the letter of the corresponding equation below each graph. 2 points for each correct answer. You do not need to show any work.

12. (a) $\begin{cases} x = 3\cos\theta, \\ y = 2\sin\theta \end{cases}$ (b) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ (c) $\dfrac{(x-2)^2}{4} + \dfrac{y^2}{4} = 1$ (d) $r = 4\sin\theta$



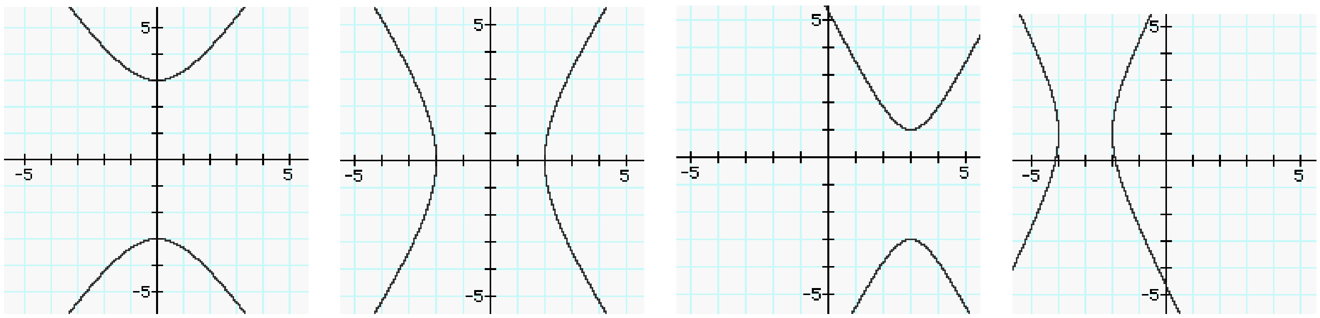






_______ _______ _______ _______

13. (a) $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$ (b) $\dfrac{(y+1)^2}{4} - (x-3)^2 = 1$ (c) $\begin{cases} x = -3 + \sec\theta, \\ y = 1 + 2\tan\theta \end{cases}$ (d) $\dfrac{y^2}{9} - \dfrac{x^2}{4} = 1$



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For problems 14 and 15 below, you must show all of your work in the space provided. Partial credit is possible on these problems. Each problem is worth 9 points.

14. Find the equation of the form $\dfrac{(x-h)^2}{p^2} + \dfrac{(y-k)^2}{q^2} = 1$ for the ellipse with center $(1, -2)$, vertex $(1, 1)$, and length of minor axis equal to 4.

15. Find all four fourth roots of the complex number $16e^{\frac{4\pi i}{3}}$. Write your answers in standard form $a + bi$, and graph the roots on the complex plane.