Math 142
Spring 2000

PRINT YOUR NAME: _____

SIGN YOUR NAME: _____

Analytic Geometry Final Exam

SECTION #: _____

For problems 1-10, show all your work, and write your answer in the blank provided. Each problem is worth 6 points. You can earn 0, 3, or 6 points on each problem. **Sufficient work must be shown to receive any credit**, and the problem must be mostly correct to earn 3 points.

1. Convert the equation $r = -2\cos\theta$ to an equation in rectangular coordinates.

1. _____

2. Find the vertex of the parabola $y^2 + 18y - 16x = -145$.

2. _____

3. Express the complex number $4e^{\frac{\pi i}{6}}$ in rectangular form.

3. _____

4. Find an appropriate first quadrant angle θ (in radians) so that a rotation of axes by θ transforms the equation $6x^2 - 3\sqrt{3}xy + 3y^2 = 100$ into a new equation of the form $au^2 + cv^2 + du + ev + f = 0$.
(You just need to find θ, *not* the new equation.)

4. _____

5. Convert the parametric equations $x = t^2 + 1$, $y = t - 1$, to an equation in x and y only.

5. _____

6. Find the cube root of $343e^{\frac{3\pi i}{4}}$ which lies in the first quadrant. Leave your answer in polar form.

6. _____

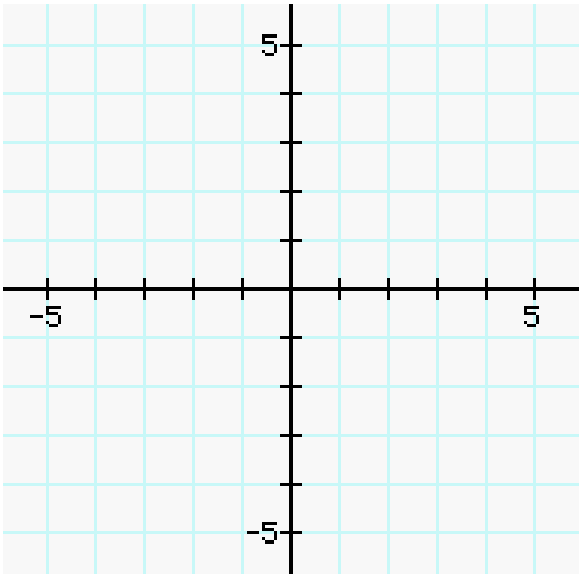
7. Find the foci of the conic section $x^2 + 4y^2 = 4$.

7. _____

8. Convert the polar coordinates $(6, \frac{2\pi}{3})$ to rectangular coordinates.

8. _____

9. A rotation of axes by the angle $\theta = \frac{\pi}{6}$ transforms the equation $-5x^2 - 6\sqrt{3}xy + y^2 - 32 = 0$ into the equation $-8u^2 + 4v^2 = 32$. Sketch this conic section, showing the rotation (i.e., draw and label the u and v axes, and draw the conic section).



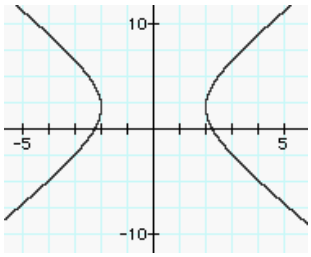
10. Find the focus of the parabola $y^2 = -20x$.

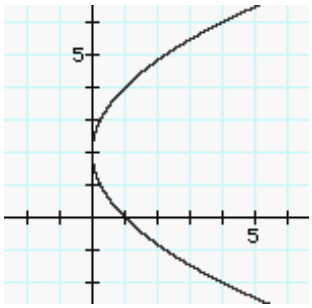
10. _____

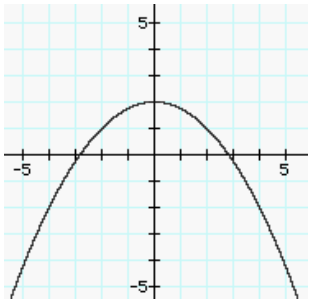
For problems 11 and 12, match the graphs with their corresponding equations. Write the letter of the corresponding equation below each graph. 2 points for each correct answer. You do not need to show any work.

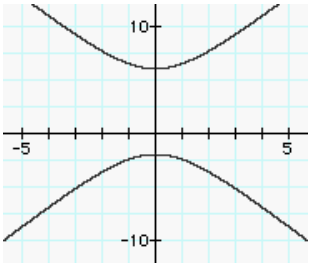
11. (a) $x^2 + 4(y-2) = 0$ (b) $(y-2)^2 - 4x^2 = 16$

(c) $4x - (y-2)^2 = 0$ (d) $\begin{cases} x = 2\sec\theta, \\ y = 2 + 4\tan\theta \end{cases}$



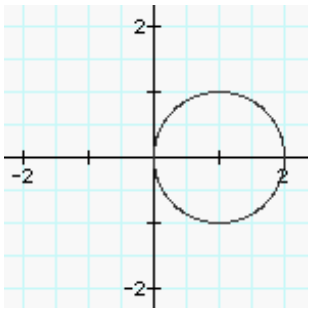


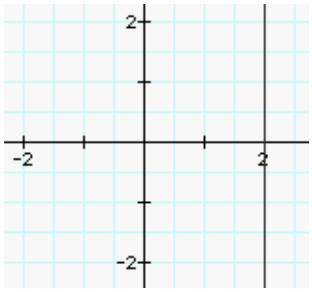


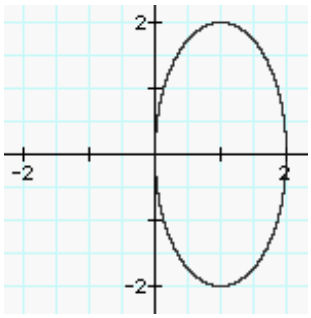


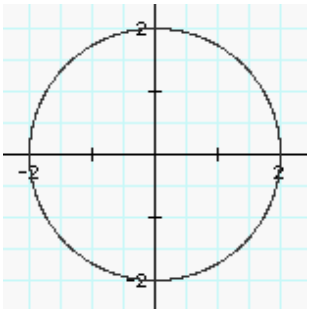
_____ _____ _____ _____

12. (a) $r = 2$ (b) $r = 2\cos\theta$ (c) $\begin{cases} x = 1 + \cos\theta, \\ y = 2\sin\theta \end{cases}$ (d) $r = \dfrac{2}{\cos\theta}$









_____ _____ _____ _____

For problems 13-15 below, you must show all of your work in the space provided. Partial credit is possible on these problems. Each problem is worth 8 points.

13. Find the equation of the form $\frac{(x-h)^2}{p^2} - \frac{(y-k)^2}{q^2} = 1$ for the hyperbola with foci $(-5, 1)$, and $(1, 1)$, and length of the conjugate axis equal to 3.

14. A satellite dish is shaped like a paraboloid. The signals that emanate from a satellite strike the surface of the dish and are reflected to a certain point where the receiver is located. If the dish is 12 feet across its opening, and is 4.5 feet deep at its center, how far from the center of the dish should the receiver be placed?

15. (a) Express the complex number $-\sqrt{3} + i$ in polar form.

(b) Compute the exact value of $(-\sqrt{3} + i)^6$ and express your answer in rectangular form.