

A. Appendix: Theorems' and Corollaries' Proofs

THEOREM 1. *Given a CFG G , a set \mathcal{E} of events, and the equivalence relation $\mathcal{R}_{\mathcal{E}}$, there is a one-to-one and onto mapping between the equivalence classes of $\mathcal{R}_{\mathcal{E}}$ and the paths of the EFG G_{EFG} where each EFG path produces the event trace corresponding to an equivalence class.*

Proof. The equivalence relation $\mathcal{R}_{\mathcal{E}}$ partitions the CFG paths into equivalence classes such that all paths in an equivalence class have the same event trace, and the CFG paths that are in different equivalence classes have different event traces.

Since G_{EFG} is the node-induced subgraph of the given CFG G consisting of the event and the relevant branch nodes, it follows that given an EFG path S , it will produce a unique event trace T and conversely given an event trace T there will be a unique EFG path S for which the event trace is T . Thus, there is a one-to-one and onto mapping between the equivalence classes of $\mathcal{R}_{\mathcal{E}}$ and the paths of the EFG G_{EFG} .

THEOREM 2. *Given a 2-event property P , its verification on all CFG paths can be done using the event traces.*

Proof. If property P holds for an object p on all CFG paths then it clearly holds for all corresponding event traces. Therefore, the case we must argue is the one in which property P is violated for object p on a CFG path S . Let T be the event trace for path S . Path S may pass through many branch nodes. We will argue that only the relevant branch nodes on that path are important in determining the existence of a feasible path with trace T . We will argue that there exists a feasible CFG path with trace T if and only if there exists a path S' with trace T that is feasible with respect to the relevant branch nodes on S .

If every path with trace T is infeasible with respect to the relevant branch nodes, then all paths equivalent to S are also infeasible, because the addition of irrelevant branch nodes cannot turn an infeasible path into a feasible one. On the other hand, suppose there exists a path S' with trace T that is feasible with respect to the relevant branch nodes. By the definition of irrelevant branch nodes, an equivalence class has paths going through all possible branches at an irrelevant branch node, so if the path S' is not feasible due to having some irrelevant branch nodes we can choose feasible branches at those nodes to construct a new CFG path that is feasible and equivalent to S' . Thus, if there exists a path S' with trace T that is feasible with respect to the relevant branch nodes on S , then there always exists a feasible CFG path with trace T .

Thus, if property P is violated on path S , then we have the following: (a) if S is feasible with respect to the relevant branch nodes on S , then there is a feasible path in the equivalence class of S , and the violation of P is a true positive; (b) if S is not feasible with respect to the relevant branch nodes on S , then all paths equivalent to S are also not feasible.

DEFINITION 1. *The boundary of a subgraph S in a directed graph G , denoted by $\text{boundary}(S)$, is the set of nodes $u \in S$ such that $\text{succ}(u) \in \text{succ}(S)$.*

THEOREM 3. *Let G be a colored T -irreducible and acyclic graph. Then for any subgraph S containing non-colored nodes of G : $|\text{succ}(S)| \geq 2$.*

Proof. If a non-colored node $u \in G$ has only one successor then it is eliminated by transformation T_1 . Thus, since G is T -irreducible, $|\text{succ}(u)| \geq 2$ for all non-colored nodes $u \in G$. Also, by assumption, G is an acyclic graph. Using these two facts, we will show that every subgraph S has a node with at least two successors outside S and thus $|\text{succ}(S)| \geq 2$.

Let $P_{v_0 \rightarrow v_n} : (v_0, v_1), (v_1, v_2), \dots, (v_{n-1}, v_n)$ be a maximal path in subgraph S . Since v_n is the terminal node of this maximal path P , its successor cannot be another node in S not on the path P . Also, the successor of v_n cannot be another node on the path P because G_c is an acyclic graph, so v_n must be a boundary node and all its successors must be outside the subgraph S . Since v_n is a non-colored node, $|\text{succ}(v_n)| \geq 2$. Since v_n , a node in S , has at least two successors outside of S , we have $|\text{succ}(S)| \geq 2$. This completes the proof.

COROLLARY 2. *Let G be a CFG and G_{cEFG} be the condensed EFG. Then, for any subgraph S containing non-colored nodes of G_{cEFG} , $|\text{succ}(S)| \geq 2$.*

Proof. Note that the condensed EFG G_{cEFG} is the graph resulting from step (4) of the EFG construction algorithm. By construction, the condensed graph G_{cEFG} is a colored T -irreducible graph. Also, by construction G_{cEFG} is an acyclic graph. By applying the above theorem to G_{cEFG} we get the proof of the corollary.

COROLLARY 3. *The graph produced by Algorithm II does not contain any irrelevant branch nodes.*

Proof. By construction, the graph $G_{\text{T-irr}}$ resulting after step (1) of Algorithm II, consists of only event nodes, relevant branch nodes, and the irrelevant branch nodes retained by Algorithm I. We will now argue that all the irrelevant branch nodes will be eliminated when G_{cEFG} is constructed in step (4) of Algorithm II. According to the definition of irrelevant branch nodes (section 3), a node c is irrelevant if there is a subgraph S that contains c , all its branch edges, S has no event nodes, and $|\text{succ}(S)| = 1$. It follows from this definition and from the corollary 2 that G_{cEFG} does not contain any irrelevant branch nodes. Thus, the final graph produced by Algorithm II also does not contain any irrelevant branch nodes, because it consists of the nodes in G_{cEFG} and all the event nodes.