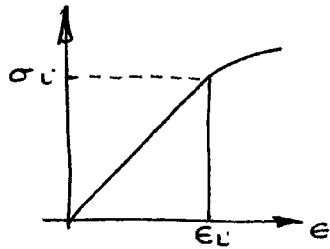


DISTORTION ENERGY



$$U = \text{STRAIN ENERGY} = \frac{1}{2} \epsilon_L \sigma_L$$

3-D

$$U = \frac{1}{2} (\sigma_1 \epsilon_1 + \sigma_2 \epsilon_2 + \sigma_3 \epsilon_3)$$

$$\epsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2 - \nu \sigma_3)$$

$$\epsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1 - \nu \sigma_3)$$

$$\epsilon_3 = \frac{1}{E} (\sigma_3 - \nu \sigma_1 - \nu \sigma_2)$$

$$U = \frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_1 \sigma_3)]$$

$$U = U_h + U_d$$

$$= \underbrace{\text{HYDROSTATIC}}_{\text{VOLUMETRIC CHANGE}} + \underbrace{\text{DISTORTION}}_{\text{MEASURES AMOUNT OF SHEAR}}$$

$$\sigma_1 = \sigma_h + \sigma_{1d}$$

$$\sigma_2 = \sigma_h + \sigma_{2d}$$

$$\sigma_3 = \sigma_h + \sigma_{3d}$$

$$\sigma_1 + \sigma_2 + \sigma_3 = 3\sigma_h + \underbrace{\sigma_{1d} + \sigma_{2d} + \sigma_{3d}}_{0 \text{ (VOLUME CHANGE - NO DISTORTION)}}$$

$$\sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$U_h = \frac{1}{2E} [\sigma_h^2 + \sigma_h^2 + \sigma_h^2 - 2\nu (\sigma_h \sigma_h + \sigma_h \sigma_h + \sigma_h \sigma_h)]$$

$$= \frac{1}{2E} [3\sigma_h^2 - 2\nu (3\sigma_h^2)]$$

$$U_h = \frac{3}{2} \frac{(1-2\nu)}{E} \sigma_h^2$$

Distortion Energy Theory:

The distortion energy theory says that failure occurs due to distortion of a part, not due to volumetric changes in the part (distortion causes shearing, but volumetric changes due not).

As examples:

Rocks below the earth's surface. Pressure exerted on them is essentially uniform and well beyond their compressive ultimate strengths, yet the rocks are able to withstand the pressure of the water without fracture. The explanation for their survival is that because the shape of the rock is not distorted, there is no shearing, therefore no failure.

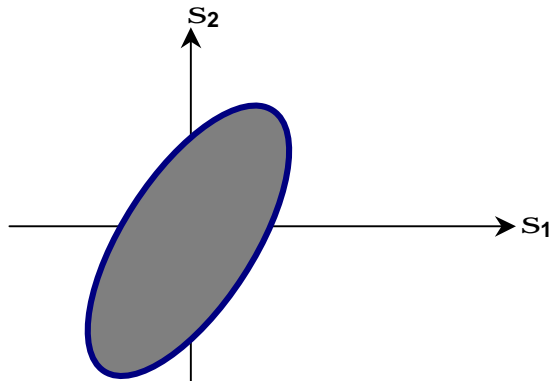
Failure will occur if

$$s' = \sqrt{s_1^2 + s_2^2 + s_3^2 - s_1s_2 - s_2s_3 - s_1s_3} \geq S_y$$

In terms of applied stresses

$$s' = \sqrt{\frac{(s_x - s_y)^2 + (s_y - s_z)^2 + (s_z - s_{xy})^2 + 6(t_{xy}^2 + t_{yz}^2 + t_{zx}^2)}{2}}$$

σ' is called the Von Mises effective stress.



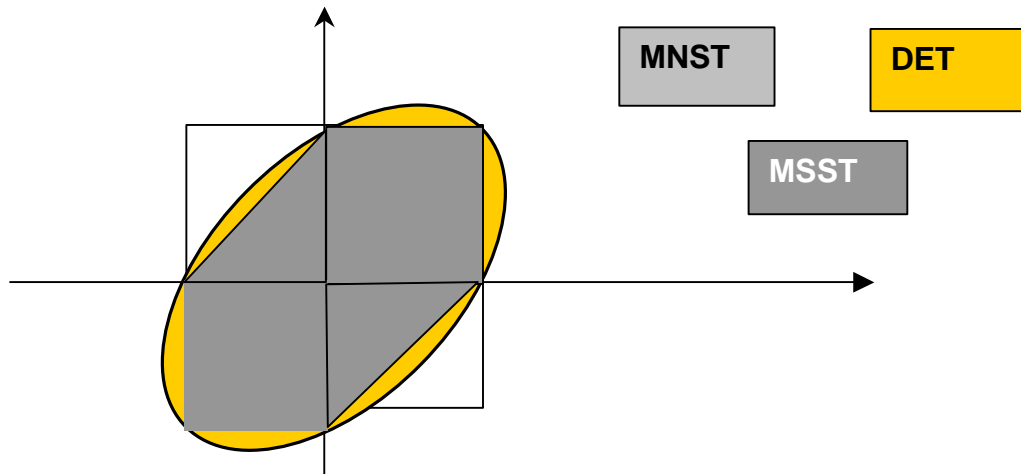
For the case of pure shear, i.e., $\sigma_1 = \tau = -\sigma_3$, $\sigma_2 = 0$

$$s_y^2 = s_1^2 - s_1s_3 + s_3^2 = 3s_1^2 = 3t_{\max}^2$$

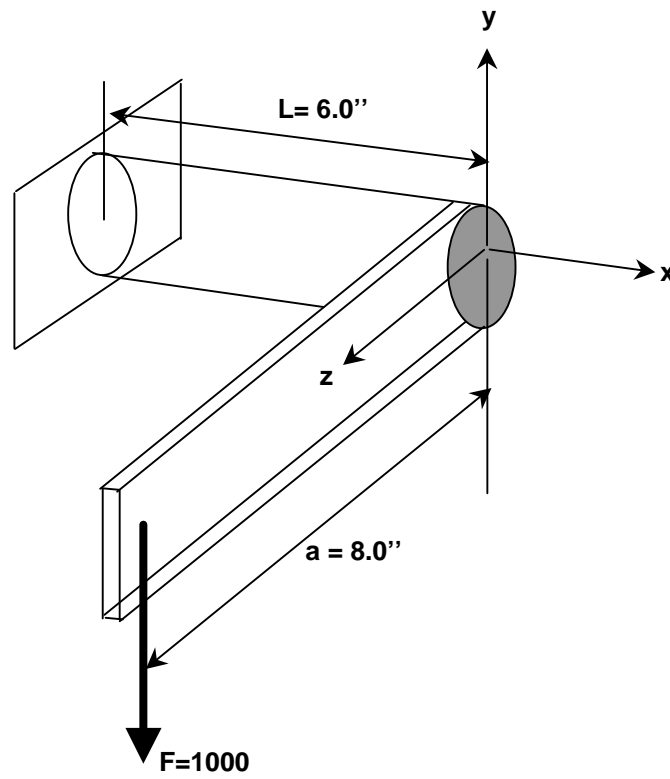
$$s_1 = \frac{s_y}{\sqrt{3}} = 0.577s_y = t_{\max}$$

Shear yield strength, $S_{sy} = 0.577 S_y$

Distortion Energy Theory is less conservative than Maximum Shear Strength Theory, but more conservative than the Maximum Normal Stress Theory



Determine whether or failure is predicted with the Distortion Energy Theory for the shaft loaded as shown



DISTORTION ENERGY

$$U_h = \frac{3}{2} \left(\frac{1-2\nu}{E} \right) \sigma_h^2$$

$$\sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$U_h = \frac{1-2\nu}{6E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3) \right]$$

$$U_d = U - U_h$$

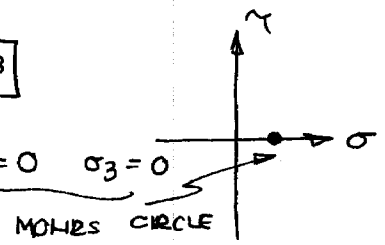
$$= \frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3) \right]$$

$$- \frac{(1-2\nu)}{6E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3) \right]$$

$$U_d = \frac{1+\nu}{3E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3 \right]$$

FOR A TENSILE TEST

$$\sigma_1 = s_y \quad \sigma_2 = 0 \quad \sigma_3 = 0$$



$$U_d = \frac{1+\nu}{3E} \left[s_y^2 \right]$$

$$\frac{1+\nu}{3E} s_y^2 = \frac{1+\nu}{3E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3 \right]$$

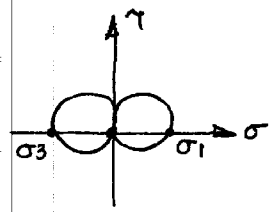
$$s_y^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3$$

$$s_y = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3}$$

VON MISES STRESS
 σ'

$$\frac{s_y}{\sigma'} = N.$$

FOR PURE SHEAR $\sigma_1 = -\sigma_3$
 $\sigma_2 = 0$



$$\sigma' = \sqrt{\sigma_1^2 + \sigma_3^2 + \sigma_1^2}$$

$$s_y = \sqrt{3\sigma_1^2}$$

$$\frac{s_y}{\sqrt{3}} = \sigma_1 = 0.577 s_y$$