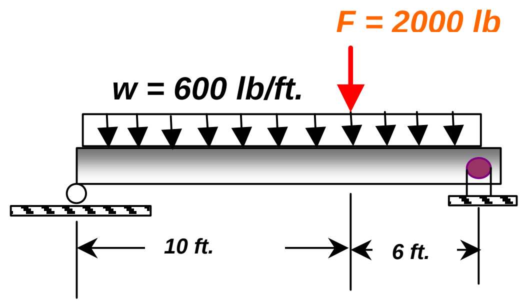
Deflections by Superposition

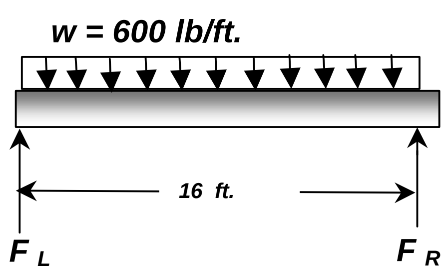
The central idea of superposition is that slopes and deflections, due to individual loads, may be added (however, it must remain true that a linear relationship exists between stresses and/or deflections and the loads causing them).

An example best demonstrates this method. Consider the following beam and its loadings. We intend to find the deflection at mid-span of this beam. Young' s modulus of elasticity, E, is 30 E6 psi, and the 2nd moment of area, I, is 180 in.^{4}

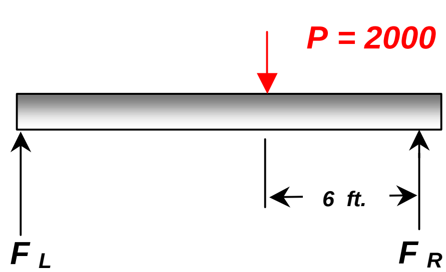


This problem can be broken down into the following two problems:

Case 1– Effect of 600 lb. per ft. load, w



Case 2– Effect of 2000 lb. load, P



Most texts that are used for Strengths of Materials courses contain tables that show common loadings along with equations for shear, slope, and deflection. You can use these tables to solve superposition problems. For the example given above, look at the following table:

Case	1 distributed load—simply supported beam	2 Point load—simply supported, load not symmetrically distributed**
Slope	$\frac{dv}{dx} = \frac{wx}{24EI}\left[L^3 - 6Lx^2 + 4x^3\right]$	$\frac{dv}{dx} = \frac{Pb}{6EIL}\left[L^2 - b^2 - x^2\right] 0 \le x \le a$
Deflection	$v = \frac{wx}{24EI}\left[L^3 - 2Lx^2 + x^3\right]$	$v = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right) 0 \le x \le a$ $v_{\max} = \frac{Pb}{9\sqrt{3}EI}\left(L^2 - b^2\right)$

***a is from the left end of the beam to the load*
***b is from the right end of the beam; when a £ x £ L, the right hand end of the beam is treated as the origin*

For case 1:

$$v = \frac{50x}{24EI}\left[(192)^3 - 2(192)x^2 + x^3\right]$$
$$v = \frac{50*96}{24*30E6*180}\left[(192)^3 - 2(192)(96)^2 + 96^3\right]$$
$$=$$
$$0.1638\ inches$$

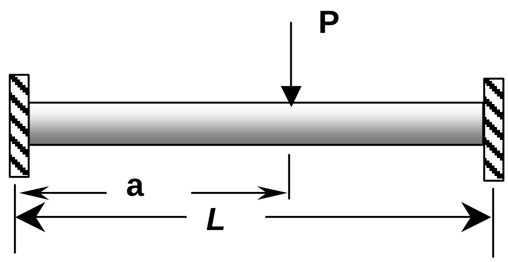
For case 2:

$$v = \frac{2000*72x}{6EIL}\left(192^2 - 72^2 - x^2\right)$$
$$= \frac{2000*72*96}{6*30E6*180*192}\left(192^2 - 72^2 - 96^2\right)$$
$$=$$
$$.04992$$

The total deflection at mid-span would be 0.1638 inches + .04992 inches = **<u>.21372 inches.</u>**

Practice Problem– Deflection by Superposition

Solve the following problem by the method of Superposition

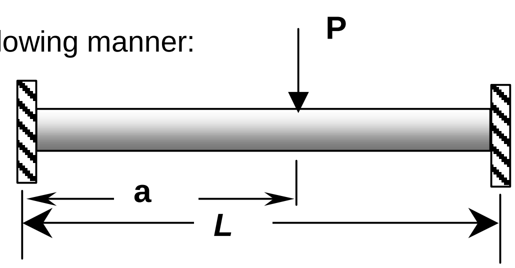


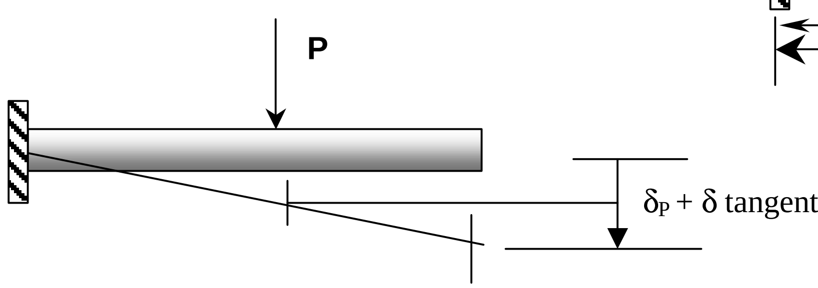
This problem is called a statically indeterminate problem. That means you have more unknown loadings than you have available equilibrium equations.

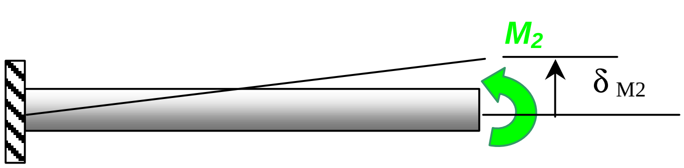
The shear forces at walls are unknown, V_1, and V_2, and there are two moments, M_1, and M_2 that are unknown. The two equilibrium equations available are:

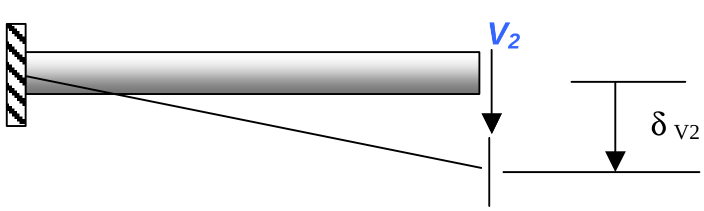
1). Summation of vertical forces
2). Summation of moments.

To help you get started think of the loaded beam in the following manner:









You can use the following 4 boundary conditions for solving for V_1, V_2, M_1, and M_2

The slope and deflection of the right end of the beam due to V_2 (two equations).
The slope and deflection of the right end of the beam due to M_2 (two equations).

You also know the following:

The deflection at the right end of the beam is 0.0 inches and the slope is zero radians.
(all slopes sum to zero and all deflections sum to zero).