BOD and Dissolved Oxygen (DO) Sag Curve

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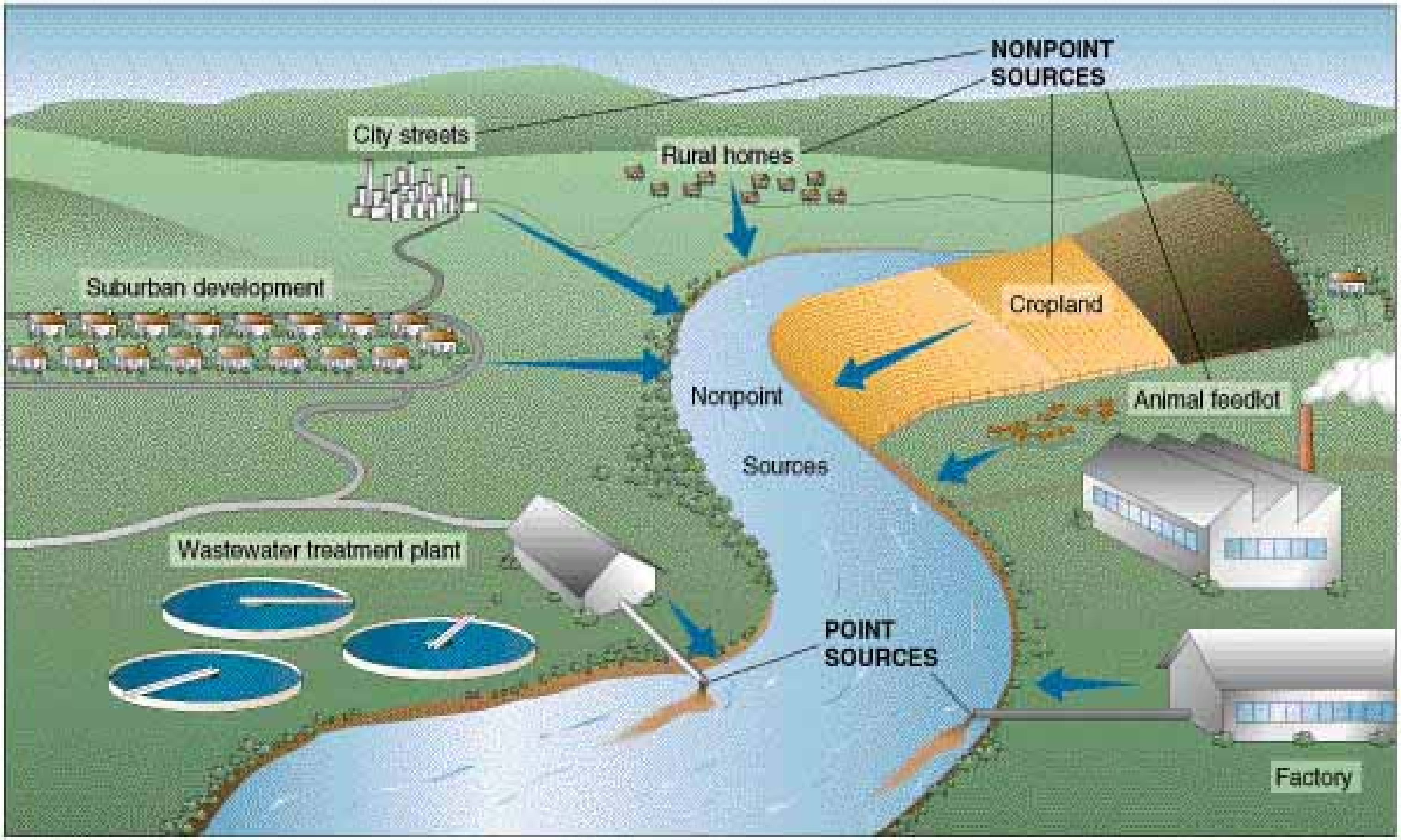
Major Pollutants and their Sources

Pollutants	Point source: Domestic sewage	Point source: Industrial wastewater	Non-point source: Agricultural runoff	Non-point source: Urban runoff
Organic matter (COD/BOD)	x	x	x	x
N & P	x	x	x	x
Sediment (SS)	x	x	x	x
Pathogens	x	x?	x	x?
Salts	-	x	x	x
Heavy metals	-	x	-	x
Toxic compounds	-	x	x	-
Thermal	-	x	-	-

Point and Non-point Source Pollutions



Organic Pollution and Oxygen Demand

Bacteria

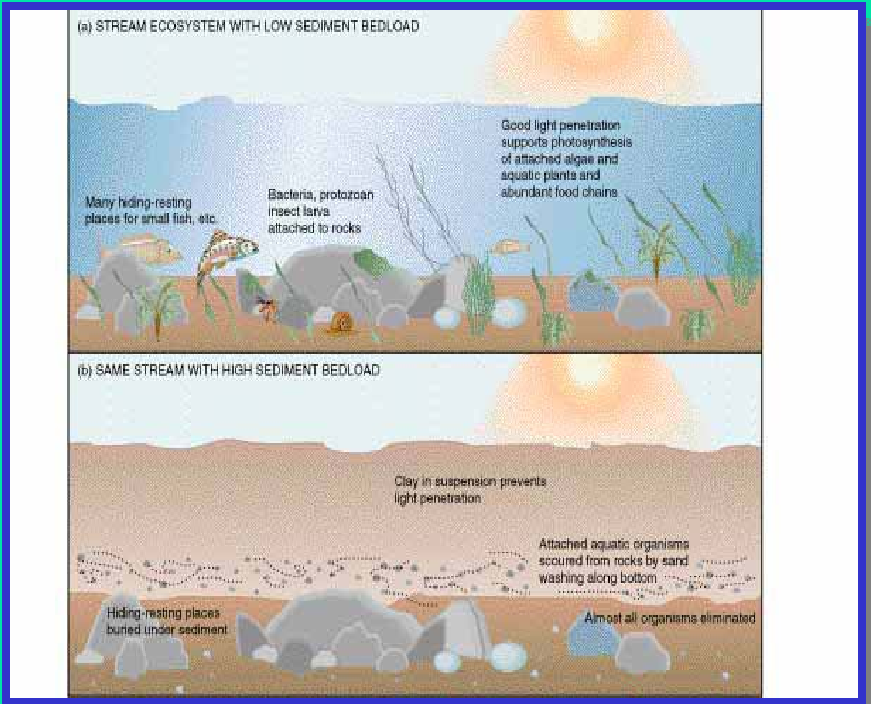
$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$$

180 g 192 g

1g glucose = 192/180g O_2

1g glucose consumes 1.07g O_2

1g glucose has BOD of 1.07g



Organic matter in waste + O_2 in stream → CO_2 + H_2O

DO level ≤ 2.0 mg/L, some fish and other aquatic animal Species may be distressed and some species may even die due to suffocation.

As a rule of thumb DO level of 4 mg/L should be maintained

Organic Pollution and Fish Kill

Slocum Creek, a tributary of the Neuse below Carolina Pines Sep. 4, 2003: North Carolina

Nitrogen Pollution and Oxygen Demand

$$NH_4^+ + 2O_2 \xrightarrow{\text{Bacteria}} NO_3 + H_2O + 2H^+$$

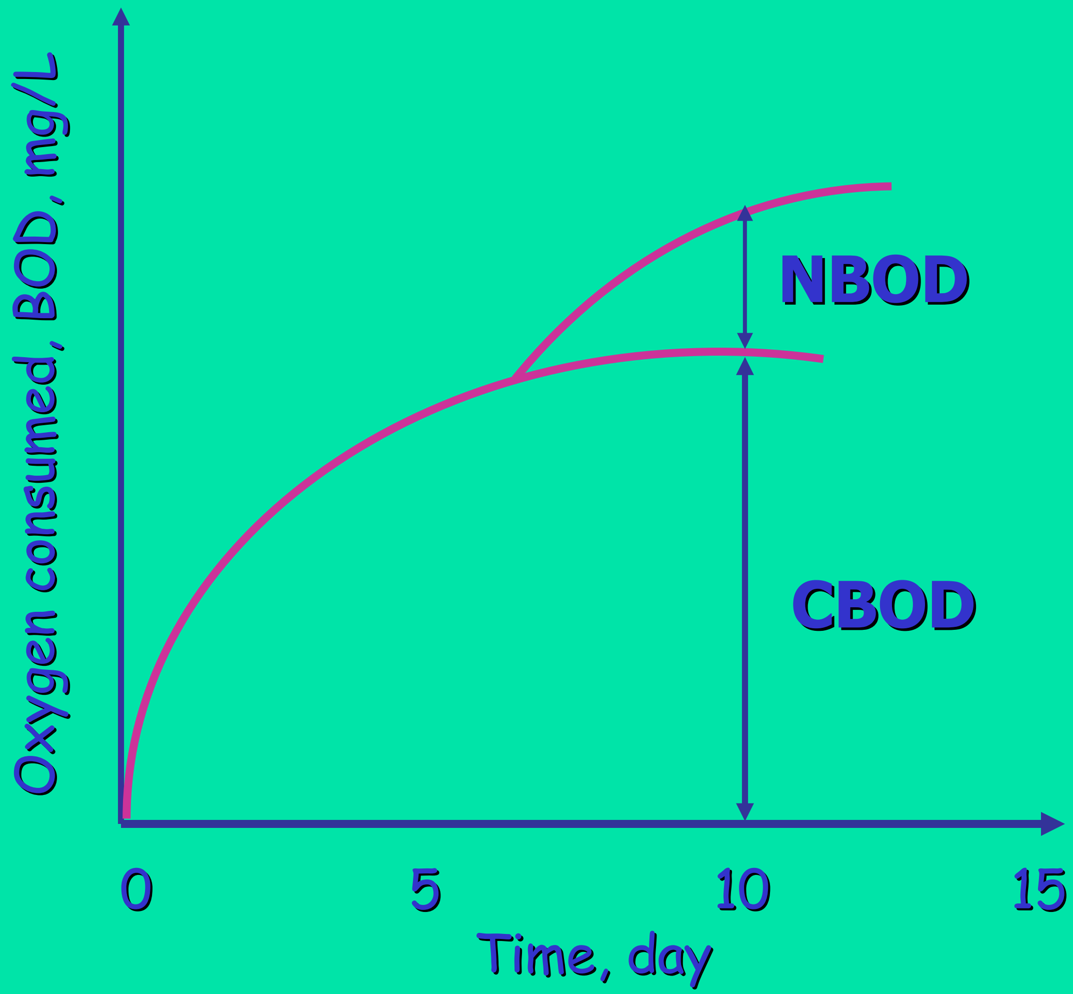
14 g (N) 64 g

1g NH_4^+-N = 64/14g O_2

1g NH_4^+-N consumes 4.57 g O_2

Fish kill in Maryland

Oxygen consumption or BOD curve



Oxygen Consumption and Biochemical Oxygen Demand (BOD)

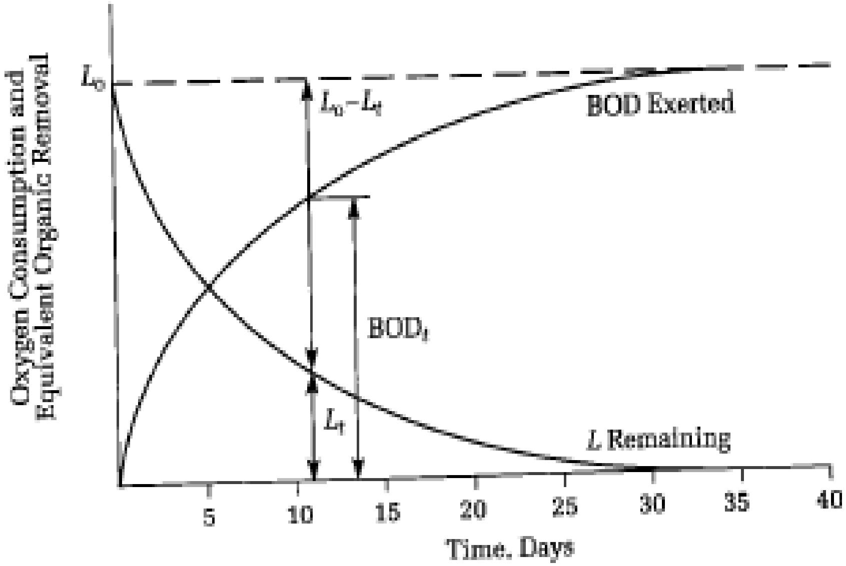


FIGURE 4-1
BOD and oxygen-equivalent relationships.

k=reaction rate constant, day^{-1}

Rate of organic removal $\propto$ Organic matter remaining any time

$$\frac{dL_t}{dt} \propto L_t$$

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$$\frac{dL_t}{dt} = -k\,L_t$$

➔

$$\frac{dL_t}{L_t} = -k\,dt$$

$$\int_{L_0}^{L} \frac{dL_t}{L_t} = -k\int_0^t dt$$

→

$$\ln \frac{L_t}{L_0} = -kt$$

$$L_t = L_0 e^{-kt}$$

Organic matter consumed, BOD (Y_t) = Lo-Lt

$$y_t = L_0 - L_0 e^{-kt}$$

$$y_t = L_0 (1 - e^{-kt})$$

Lo = Ultimate BOD
Usually BOD test is conducted in 5 days.

Temperature correction: $k_T = k_{20}\, \theta^{T-20}$

$\theta = 1.047$

The BOD_5 of a wastewater is determined to be 150 mg/L at 20°C. The k value is 0.23 day^{-1}. What would be the BOD_8 If the test were run at 15°C.

1. Determine ultimate BOD (Lo):

$$y_5 = L_0(1 - e^{-kt}) \rightarrow L_0 = \frac{y_5}{1 - e^{-kt}}$$

$$L_0 = \frac{150}{1 - e^{-0.23x5}}$$ = 220 mg/L

2. Temperature correction for k value for 15°C:

$k_T = k_{20}\,\theta^{T-20} \rightarrow k_{15} = 0.23\,(1.047)^{15-20} = 0.18\ day^{-1}$

3. Determine BOD at 8 days, y_8:

$$y_8 = L_0(1 - e^{-kt}) \rightarrow y_8 = 220(1 - e^{-0.18x8})$$ = 168 mg/L

Combined BOD and DO determination for a stream

- Combined waste flow

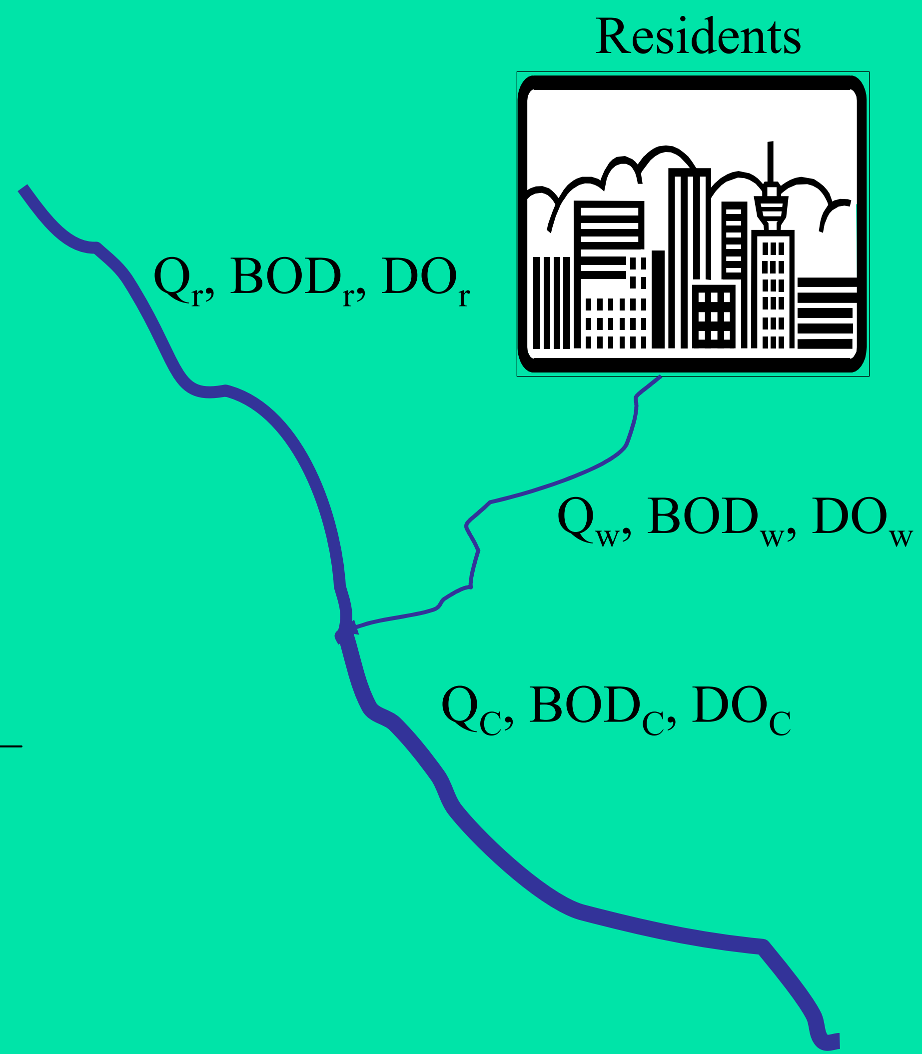
$$Q_C = Q_r + Q_w$$

- Combined BOD

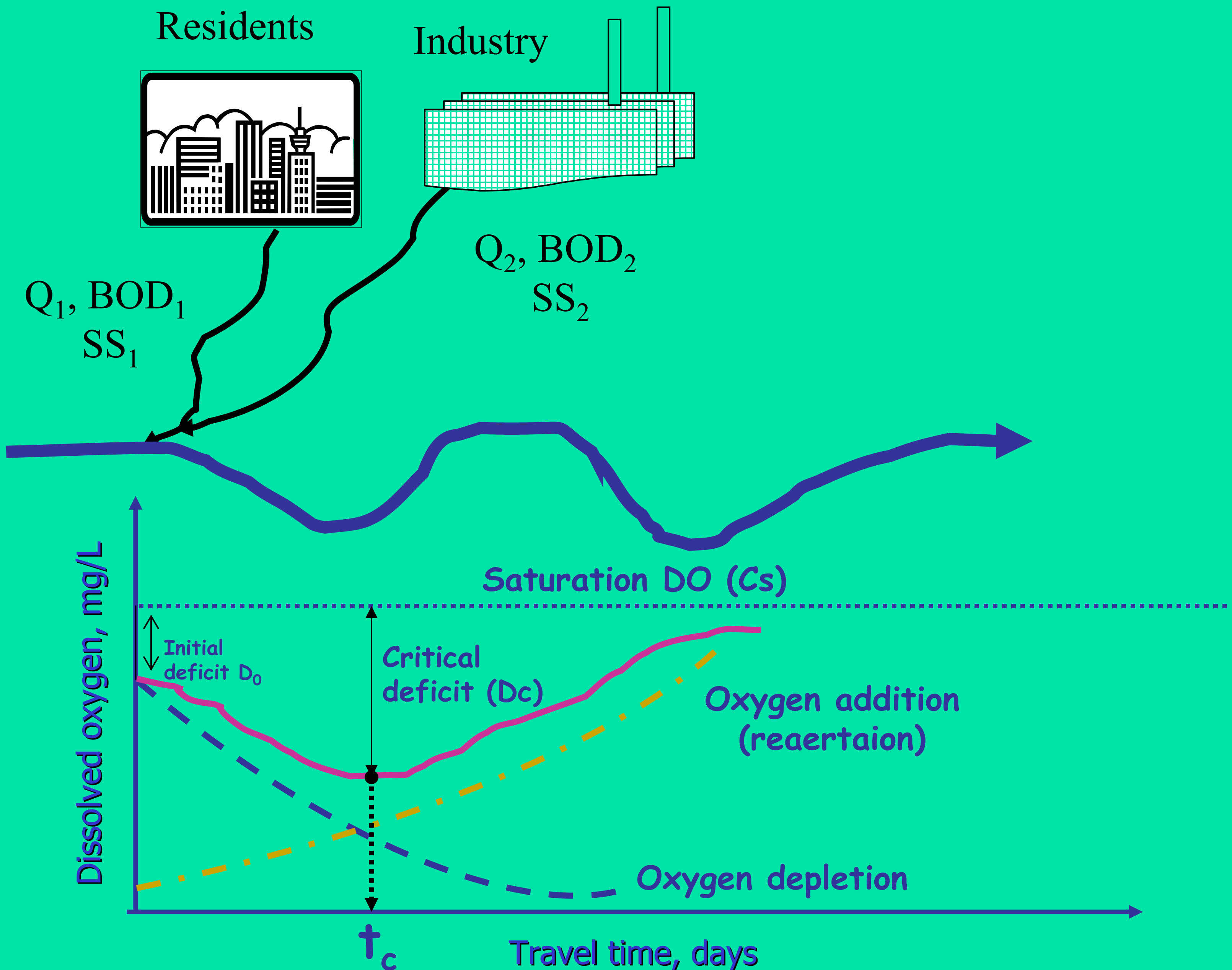
$$BOD_C = \frac{Q_r BOD_r + Q_w BOD_w}{Q_1 + Q_2}$$

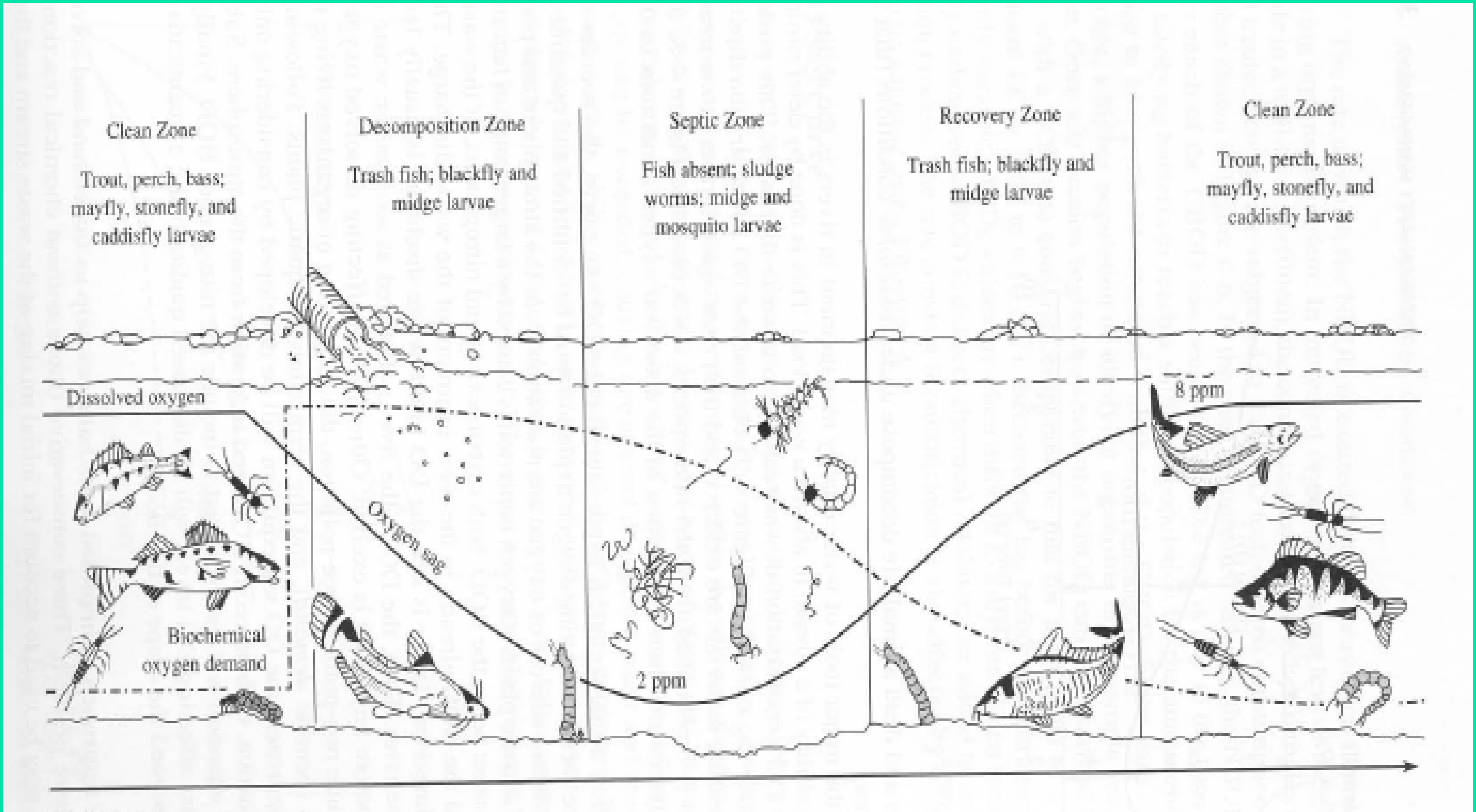
- Combined DO

$$DO_c = \frac{Q_r{*}DO_r + Q_w{*}DO_w}{Q_1 + Q_2}$$



DO Sag Curve


Residents
Industry
Q_1, BOD_1
SS_1
Q_2, BOD_2
SS_2
Saturation DO (Cs)
Initial deficit D_0
Critical deficit (Dc)
Oxygen addition (reaertaion)
Oxygen depletion
Dissolved oxygen, mg/L
t_c
Travel time, days



Clean Zone
Trout, perch, bass; mayfly, stonefly, and caddisfly larvae
Decomposition Zone
Trash fish; blackfly and midge larvae
Septic Zone
Fish absent; sludge worms; midge and mosquito larvae
Recovery Zone
Trash fish; blackfly and midge larvae
Clean Zone
Trout, perch, bass; mayfly, stonefly, and caddisfly larvae
Dissolved oxygen
Oxygen sag
Biochemical oxygen demand
2 ppm
8 ppm

Oxygen deficit is a function of oxygen consumption and oxygen supply by reaeration

$$\frac{dD}{dt} = k_d(L_t) - k_r(D)$$

dD/dt = Rate of change of oxygen deficit
kd = Deoxygenation rate constant
Lt = BOD at any time t
kr = Reaeration rate constant
D = Oxygen deficit (combined w/w and river)

Integrate and solve above equation for D:

$$\boxed{D = \frac{k_d L_o}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_0 e^{-k_r t}}$$

Critical Deficit, Dc

$$D_c = \frac{k_d}{k_r} L_0 e^{-k_d t_c}$$

Critical Time, tc

$$t_c = \frac{1}{k_r - k_d} \ln\left[\frac{k_r}{k_d}\left(1 - D_0 \frac{k_r - k_d}{k_d L_o}\right)\right]$$

Example:

Find DO concentration 50 km downstream from a discharge with the following characteristics:

	wastewater	River
Flow, m^3/s	0.05	0.5
Ultimate BOD (L_o), mg/L	50	10
DO, mg/L	1	6
k_d, day^{-1}		0.16
k_r, day^{-1}		0.18
Velocity, m/s		0.1
Temperature, °C	25	25

DO_{sat} @ 25°C = 8.38 mg/L

1. Find time (t) to travel 50 km:

t = distance/velocity = (50,000 m)/(0.1m/sec) = 5.78 day

2. Calculate combined ultimate BOD of w/w and river

Lo = [0.05 (50) + 0.5 (10)]/(0.05 +0.5) = 13.64 mg/L

3. Calculate DO after mixing of w/w and river

$DO_{combined}$ = [0.05(1) +0.5 (6)]/(0.5+0.05) = 5.55 mg/L

Initial deficit (Do) = 8.38-5.55 = 2.83 mg/L

4. Calculate deficit at t=5.78 day,

$$D = \frac{k_d L_o}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_0 e^{-k_r t}$$

Deficit at t = 5.78 days = 5.725 mg/L

Therefore, DO conc. At 5.78 day will be:

DO = Dsat - Dt = 8.38-5.725 = 2.66 mg/L

Your assignment: calculate tc and Dc

$$t_c = \frac{1}{k_r - k_d} \ln\left[\frac{k_r}{k_d}\left(1 - D_0 \frac{k_r - k_d}{k_d L_o}\right)\right]$$

$$D_c = \frac{k_d}{k_r} L_0 e^{-k_d t_c}$$